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The above solution is in the real factors of z^4-2z^2+2.

On page 140, No. 5, Vol. IX, Dr. Zerr gives a solution, using the imaginary factors of z^4-2z^2+2. His final seemingly imaginary result reduces to a real value by substituting $\sqrt{\frac{1+\sqrt{2}}{2}}+\sqrt{\frac{1-\sqrt{2}}{2}}$ for $\sqrt{[1+\sqrt{(-1)}]}$ and $\sqrt{\frac{1+\sqrt{2}}{2}}-\sqrt{\frac{1-\sqrt{2}}{2}}$ for $\sqrt{[1-\sqrt{(-1)}]}$, reducing and remembering that

$$\log[a+b\sqrt{(-1)}]=\sqrt{(-1)}\tan^{-1}\frac{b}{a}+\log\sqrt{(a^2+b^2)}.$$

216. **Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.**

Find the limit of the sum of the series

$$\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\dots\dots+\frac{n}{n^2+m^2},$$

when n and m are indefinitely increased. (Distinguish the several cases arising from the different *relative* values of m and n.)

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\dots\dots+\frac{n}{n^2+m^2}$$

$$=h\left[1+\frac{1}{1+h^2}+\frac{1}{1+(2h)^2}+\frac{1}{1+(3h)^2}+\dots\dots+\frac{1}{1+(mh)^2}\right]$$

where $h=\frac{1}{n}$. Hence the required limit is $\int_0^{mh}\frac{dx}{1+x^2}=\tan^{-1}(mh)$.

If m is small compared with n so that $mh=0$, the limit is $\tan^{-1}0=0$. If m is equal to n so that $mh=1$, the limit is $\tan^{-1}1=\frac{1}{4}\pi$. If m is large compared with n so that $mh=\infty$, the limit is $\tan^{-1}\infty=\frac{1}{2}\pi$.

Also solved by S. A. Corey, and Henry Heaton.

217. **Proposed by S. A. COREY, Hiteman, Iowa.**

In *The Analyst*, Vol. II, p. 120, 1875, Dr. G. W. Hill finds by the method of mechanical quadrature the value of $\int_0^{\frac{1}{2}\pi}\frac{xdx}{\sin x[1+.16\cos^2x]^{\frac{3}{2}}}$ to be 1.6576363. Evaluate the definite integral by some other method and verify above result.

Solution by the PROPOSER.

Using the formula* $f(x)-f(0)=\frac{x}{m.2}\left\{[f'(x)+f'(0)]+2\left[f'\left(\frac{x}{m}\right)\right.\right.$

MONTHLY, Vol. 13, p. 12.

$$+f'\left(\frac{2x}{m}\right)+f'\left(\frac{3x}{m}\right)+\dots\dots+f'\left(\frac{m-1}{m}x\right)\Big]\Big\}-\frac{B_1x^2}{m^2.2!}[f''(x)-f''(0)]$$

$$+\frac{B_2x^4}{m^4.4!}[f^{\mathrm{iv}}(x)-f^{\mathrm{iv}}(0)]-\dots\dots+(-1)^n\frac{B_nx^{2n}}{m^{2n}.(2n)!}[f^{(2n)}(x)-f^{(2n)}(0)]+\dots\dots$$

(B_1, B_2, ..., B_n being Bernoulli's numbers, $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{42}$, etc.) and taking $m=6$, we get

$$\tfrac{1}{24}\pi f'(\tfrac{1}{2}\pi)=\pi^2/48= \quad . \quad . \quad . \quad . \quad . \quad . \qquad .205,616,758$$

$$\tfrac{1}{24}\pi f'(0)=\frac{\pi}{24(1.16)^{\frac{3}{2}}}= \quad . \quad . \quad . \quad . \quad . \qquad .104,773,547$$

$$\tfrac{1}{12}\pi f'(\tfrac{1}{12}\pi)=\frac{\pi^2}{144\sin 15°(1+.16\cos^2 15°)^{\frac{3}{2}}}= \quad . \quad . \quad . \qquad .214,932,009$$

$$\tfrac{1}{12}\pi f'(\tfrac{1}{6}\pi)= \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad .231,297,112$$

$$\tfrac{1}{12}\pi f'(\tfrac{1}{4}\pi)= \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad .259,082,377$$

$$\tfrac{1}{12}\pi f'(\tfrac{1}{3}\pi)= \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad .298,480,940$$

$$\tfrac{1}{12}\pi f'(\tfrac{5}{12}\pi)= \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad .349,155,189$$

$$\text{Sum}=1.663,337,932$$

$$\frac{B_1(\frac{1}{2}\pi)^2}{6^2.2!}[f''(\tfrac{1}{2}\pi)-f''(0)]=\frac{\pi^2}{1728}= \quad . \quad . \quad . \quad . \qquad .005,711,577$$

$$1.657,626,355$$

$$\frac{B_2(\frac{1}{2}\pi)^4}{6^4.4!}[f^{\mathrm{iv}}(\tfrac{1}{2}\pi)-f^{\mathrm{iv}}(0)]=\frac{\pi^4}{30.12^4.4!}[3-9(.16)]=\frac{156\pi^4}{3000.12^4.4!}= \qquad .000,010,178$$

$$1.657,636,533$$

$$\frac{B_3(\frac{1}{2}\pi)^6}{6^6.6!}[f^{\mathrm{vi}}(\tfrac{1}{2}\pi)-f^{\mathrm{vi}}(0)]=\frac{\pi^6}{42.12^6.6!}[225(.16)^2-30(.16)+25]$$

$$=\frac{2596\pi^6}{4200.12^6.6!}= \quad . \quad . \quad . \quad . \quad . \qquad .000,000,009$$

Value of integral correct to eight or 9 decimal places . 1.657,636,524
Value of integral obtained by Hill 1.657,636,3

As Hill used but seven decimal places in his computations we would not expect his result to be correct to the last decimal place. In the foregoing computations the term involving B_3 might have been omitted and the result would still have been more accurate than Hill's result, although his method involves somewhat more labor than does the foregoing.

219. Proposed by C. N. SCHMALL, College of the City of New York, New York City.

In the article "Infinitesimal Calculus" in the *Encyclopaedia Britannica* Vol. XIII, page 24, I notice the following: "Of all triangular pyramids standing on a given triangular base, and of given altitude, find that whose surface is the least." A solution is required.